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A Decision Support System using Probability Hypothesis Density Filter and Hungarian Optimization Algorithm

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Abstract

One of the major challenges for aeronautical safety system is to prevent breakdown and impact of malfunctioned airborne objects especially the unmanned aerial vehicles in the civil area. The safety personnel may use projectiles to dissociate such kind of object to save the civil lives and properties from destruction. The safety personnel can utilize their projectiles effectively with the help of a Decision Support System which is proposed in this article. The Probability Hypothesis Density filter, First Order Gradient based optimization and Hungarian algorithm are exploited to build the system. The result proves the efficacy of proposed system.

Keywords:

PHD filter;
First Order Gradient based optimization;
Hungarian algorithm;
Probabilistic Data fusion;
Parameter estimation.

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1. Introduction

In recent era use of unmanned aerial vehicles in civil applications becomes very popular. Its use is rapidly expanding to commercial, scientific, recreational, agricultural, and other applications, such as policing, peacekeeping and surveillance, product deliveries, aerial photography, agriculture and drone racing. However, aeronautical safety concern from airborne objects is very much essential for present day especially for unmanned aerial vehicles. Malfunctioning of such kind of airborne objects may causes harm to the civil society since it may hit and destroy the civil properties and may endanger human life. To stop a misbehaving airborne object which is uncontrollable to its operator the safety personnel may use projectiles to intercept and dissociate the objects. The number of misbehaving airborne object may vary time to time and also the numbers of available projectiles may be limited for a particular situation. Therefore the available projectiles should be utilized in an intelligent way, so that their effectiveness should be maximized. The assignment of particular projectile to dissociate a particular airborne object is generally a difficult optimization problem and is modelled as a Decision Support System. In this article we present the design concept of such type of decision support system.

The most common sensor used to detect and track an airborne object is radar. A radar senses a flying airborne object when it falls under its inspection range. In the proposed method we assume that the radar delivers the position and velocity of an airborne object in a rate of fixed time interval. The position and velocity data of a detected object at a particular time are unitedly known as the state vector or measurement vector of that object. The position and velocity data comprise 3D coordinates of position and three dimensional velocity components of the airborne object at a particular time with respect to ECEF (earth

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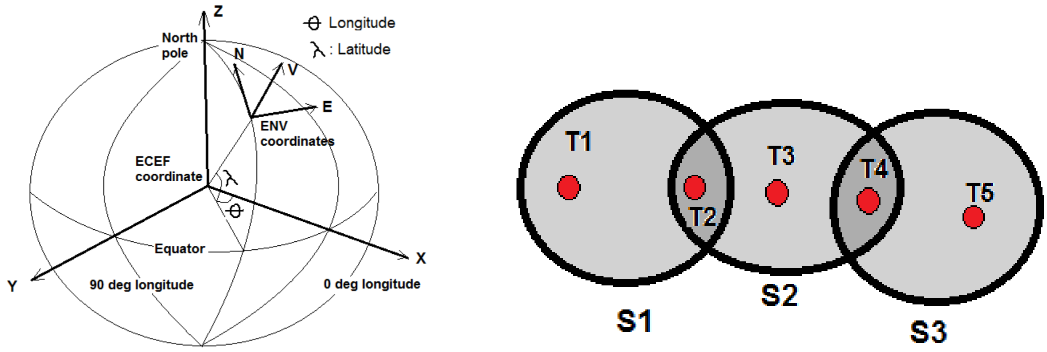
centered earth fixed) or ENV (east north vertical) reference frames as define in Figure 1(a). A sensor can detect multiple airborne objects if they come under the sensing zone of the sensor. An airborne object can also be detected by multiple sensors if it comes under the sensing zones of the sensors. In the latter case, for a single airborne object we may get multiple measurement vectors delivered by multiple sensors. In general, measurement vectors delivered by multiple sensors are corrupted by inter sensor noise and therefore may vary in terms of their values with respect to each other. The scenario becomes more complicated when multiple objects are detected by multiple sensors and hence for each object we get multiple measurement vectors. The challenge is to partition measurement vectors delivered by different sensors into different groups where each group represents a set of measurement vectors of a unique airborne object delivered by the multiple sensors at a particular time instant. We can find out the actual position of the object from its multiple measurement data utilizing tools like Probability Hypothesis Density Filter [1].



(a) (b)

Figure 1. (a): Definition of ECEF and ENV coordinate system with respect to earth center and earth surface, respectively, (b): Multiple airborne objects detection by multiple sensors.

Our proposed decision support system comprises of five main modules referred as data fusion module, airborne object trajectory prediction module, reaction time calculation module and projectile assignment module. The functionality of the data fusion module is to get a unique trajectory data of an airborne object from a set of given trajectories of the airborne object tracked by the different sensors. In our proposed method we use Probability Hypothesis Density Filter to fuse the different sensor data for getting the actual trajectory data of a airborne object and also to estimate the thrust acceleration of the airborne object to predict its complete trajectory. The airborne object trajectory prediction module next calculates the complete trajectory of the airborne object with the help of ballistic missile dynamics and the thrust acceleration calculated by its preceding module. The complete trajectory prediction of the airborne objects are essential for assessment of the impact points of the airborne objects and also crucial for finding out the interception points for destroying the airborne objects. The reaction time calculation module subsequently evaluates the reaction time of the projectiles so that a single or multiple projectiles can be assigned to destroy the airborne object. The reaction time of a projectile is the total time for taking preparation to launch a projectile and the time of travel of the projectile from launch point to the airborne object. The time of travel of the projectile from launch point to the airborne object is approximately calculated by evaluating the optimal trajectory of the projectile from launch point to its intercepting point with the airborne object exploiting the first-order gradient based optimization technique. Once the reaction time of the projectile to each airborne object is calculated, the projectile assignment module assigns the projectiles to the misbehaving airborne objects utilizing the Hungarian algorithm as an optimization tool.

The proposed technique is presented next. The result generated by the proposed decision support system is given in Section 3 followed by the conclusion in Section 4.

2. Research Method

In the proposed method, it is assumed that a sensor detects multiple objects. The maximum number of objects detected by a single sensor is denoted by m. The state of i th detected object by the sensor at a particular time instant is specified by a measurement vector denoted as $\mathbb{Z}_i$, which is equal to $[\,x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i]'$. In $\mathbb{Z}_i$, x_i, y_i and z_i represent position of the airborne object while $\dot{x}_i$, $\dot{y}_i$ and $\dot{z}_i$ represent orthogonal components of velocity of the i th airborne object in three orthogonal directions. We assume that all coordinates that sensors delivered is in ENV reference frame [2]. First, each airborne object data is tracked by fusing different sensor data and also the acceleration of the airborne object is estimated. Tracking and fusion of sensor data is performed using probability hypothesis density filter [1]. In the next step based on the parameter estimated the trajectories of the airborne objects are predicted. The airborne object tracking, fusion and parameter estimation module is described next.

2.1 Data Fusion Module

Receiving of multiple airborne objects data via multiple sensors can be explained with the help of Figure. 1(b). In Figure. 1(b), *S1*, *S2* and *S3* denote three sensors and *T1*, ..., *T5* denote five airborne objects. Sensor *S1* detects the airborne objects *T1* and *T2* and delivers measurements $\mathbb{Z}_{1,1}$ and $\mathbb{Z}_{1,2}$, respectively. Sensor *S2* detects the airborne object *T2*, *T3* and *T4* and delivers the measurement $\mathbb{Z}_{2,1}$, $\mathbb{Z}_{2,2}$and $\mathbb{Z}_{2,3}$, respectively. Similarly, sensor *S3* detects the airborne objects *T4* and *T5* and delivers the measurement $\mathbb{Z}_{3,1}$ and $\mathbb{Z}_{3,2}$, respectively. The tusk of the proposed algorithm to divide the measurements $\mathbb{Z}_{1,1}$, ..., $\mathbb{Z}_{3,2}$ into partitions $\{\mathbb{Z}_{1,1},\}$, $\{\mathbb{Z}_{1,2}, \mathbb{Z}_{2,1}\}$, $\{\mathbb{Z}_{2,2}\}$, $\{ \mathbb{Z}_{2,3}, \mathbb{Z}_{3,1}\}$, and $\mathbb{Z}_{3,2}$ corresponding to the the airborne objects *T1*, ..., *T5*, respectively.

Kalman filter [3] is a popular probabilistic data fusion method which generally is used to track airborne objects. To solve our problem we exploit PHD filter [1] which is an agglomerate of multiple Kalman Filters worked under a Gaussian mixture density canopy. The weights of the component distribution of mixture tail the probable existence of objects that are being tracked. PHD filter is more suitable to tackle multiple airborne objects scenario. The kernel of the Kalman filters embedded in the PHD filter is designed using dynamics of projectile assuming that the most of the airborne objects follow that.

Dynamics of the airborne object can be given by the set of equations shown below [3],

$$\begin{aligned} \dot{\mathrm{v}} &= \frac{\mathrm{T}\cos\alpha - \mathrm{D}}{\mathrm{m}} - \mathrm{g}\sin\gamma \\ \dot{\gamma} &= \frac{\mathrm{T}\sin\alpha + \mathrm{L}}{\mathrm{mv}} - \frac{\mathrm{g}}{\mathrm{v}}\cos\gamma \\ \dot{\mathrm{x}} &= \mathrm{v}\cos\gamma \\ \dot{\mathrm{z}} &= \mathrm{v}\sin\gamma \end{aligned} \tag{1}$$

Where, v is velocity of the airborne object, γ is the flight pah angle, T is thrust, α is angle of attack, m is the mass of airborne object, D is drag force, L is the lift force, g is the gravitational acleration, x is down range coordinate, and z is elevation of the airborne object. We track the airborne objects based on the velocity v and the flight path angle γ assuming that the cross range deviation of the airborne object is small compared to its down range. v and γ is calculated as follows,

$$\begin{aligned} \mathrm{v} &= \sqrt{\dot{\mathrm{x}}^2 + \dot{\mathrm{z}}^2} \\ \gamma &= \tan^{-1}\frac{\dot{\mathrm{z}}}{\dot{\mathrm{x}}} \end{aligned} \tag{2}$$

The states of the airborne objects are governed by continuous-time nonlinear dynamics. However the measurements are obtained at discrete instance of time. So Extended Hybrid Kalman filters [2] are used to track this type of airborne objects. The state equation of Extended Hybrid Kalman filters are calculated as follows,

$$\begin{bmatrix} \dot{\mathrm{v}} \\ \dot{\gamma} \\ \dot{\mathrm{a}} \end{bmatrix} \equiv \dot{\mathrm{X}} = \begin{bmatrix} \mathrm{a} - \mathrm{g}\sin\gamma \\ -\frac{\mathrm{g}\cos\gamma}{\mathrm{v}} \\ \mathrm{w} \end{bmatrix} = \mathrm{f}(\mathrm{X}, \mathrm{w}) \tag{3}$$

$$\mathrm{Q} = [\mathrm{w}]$$

Where, X is the State vector of the system. w is artificial noise. f is state transition function and $\dot{a}$ is estimated thrust. For Extended Kalman filter the State transition equations can be given by,

$$F = \frac{\partial f}{\partial X} = \begin{bmatrix} 0 & -g\sin\gamma & 1 \\ \frac{g cos\gamma}{v^2} & \frac{g sin\gamma}{v} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$L = \frac{\partial f}{\partial w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Similarly Measurement equation of the Kalman filter is given by,

$$Z = HX + R \tag{5}$$

Where, H and R can be defined as,

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{6}$$

$$R = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_\gamma^2 \end{bmatrix}$$

In (5) and (6) Z is the Measurement vector obtained from sensor, H is state vector to measurement vector transition matrix, R is covariance matrix corresponding to measurement error. Using (4), (5) and (6) the update equations for the Kalman filter can be written as,

$$\dot{X} = f(X, 0) + K(Z - HX) \tag{7}$$

$$K = PH'R^{-1}$$

$\dot{P} = FP + PF' + LQL' - PH'R^{-1}HP$

The update equations in (7) are obtained from the least square optimization of the measurement vectors. The equations (5)-(7) are then fused to EK-PHD filter described in TABLE IV of [1]. Main objective for exploiting EK-PHD filter is to find out number of airborne objects and their acceleration which are used for trajectory prediction of the airborne objects. We describe it into the next section.

2.2 Trajectory Prediction Module

Once we deduce a unique airborne object location we next check the path of the airborne object for a certain amount of time interval ***t***. After *t* sec if the path of the airborne object is deviated from its scheduled path, we assume that the airborne object is malfunctioning. Otherwise we assume that the airborne object is in its correct path.

Once we find that the airborne object is malfunctioning we predict the rest of the trajectory of the airborne object from following the equation derived from (1),

$$\dot{v} = -g \sin\gamma, \dot{\gamma} = -\frac{g}{v}\cos\gamma \qquad (8)$$

$$\dot{x} = v\cos\gamma, \dot{z} = v\sin\gamma$$

The integrations to find out x and z in (9) are performed following Runge-Kutta 4th order method [4]. Complete trajectory prediction of the object is essential since it helps to find out the probable area where the object may hit the ground. The cause to evaluate trajectory is to find out the available reaction time to intercept the object. We discuss it in the next section.

2.3 Reaction Time Calculation Module

After finding out the trajectory of the airborne object we find out the reaction time of the projectile so that the projectile should be selected on the basis of its capability to reach to the airborne object within the time of detection of airborne object and reaching of the airborne object to the location where the airborne object is planned to be intercepted. The reaction time of an interceptor is the total time for taking preparation to launch a projectile and the time of travel of the projectile from launch point to the airborne object. The preparation time to launch a projectile varies depending on the type of the projectile and is supplied by the domain knowledge experts.

The time of travel of the projectile from launch point to the airborne object is approximately calculated by evaluating the optimal trajectory of the projectile from launch point to its intercepting point with the airborne object. In proposed method optimal trajectory is calculated with first-order gradient based optimization of the miss distance by changing the angle of attack described in the ballistic projectile prediction equation. Miss Distance can be defined as,

$$MD = \sqrt{\left(x - x_f\right)^2 + \left(z - z_f\right)^2} \qquad (9)$$

Where, x_f and z_f are the coordinates of the intercepting point. The trajectory prediction is given in algorithm form as follows,

While $MD^{t-1} > MD^t$
//Prediction Step
$\alpha^t = \alpha^t + \Delta t\frac{\partial MD}{\partial \alpha}$ (10)
Compute x^t and y^t
From projectile equation (1); //correction step
Compute $MD = \sqrt{\left(x^t - x_f\right)^2 + \left(z^t - z_f\right)^2}$
End While

Following above algorithm the path and reaction time of the projectile calculated. Next we do the projectile airborne object pairing to assign a particular projectile to a particular airborne object. We described it in the next section.

2.4 Projectile Assignment Module

At any point of time the number of projectiles available may be limited compare to the number of airborne objects. So projectiles are to be utilized in such a way that amount of devastation should be minimized. Some of the criteria which may become crucial in time of shortage of the projectiles are mentioned below,

1. Reaction time of the projectile for a particular airborne object.

2. Distance between the launch point of the projectile to the intercepting point of the projectile and airborne object.
3. The angle between trajectory of airborne object and the projectile.
4. The importance of the zone where the impact point of the airborne object occurs.

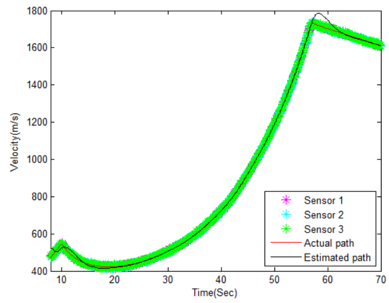
To optimize the mentioned criteria a cost function is defined. Optimization of the function will give the desired projectile airborne object pairing. The cost function is defined as follows,

$$C_{i,j} = \frac{D_{i,j}}{Z_i[1-H(\vec{d}_i\vec{d}_j)][1+H(T_w-T_t-T_{wp})]}, \tag{11}$$

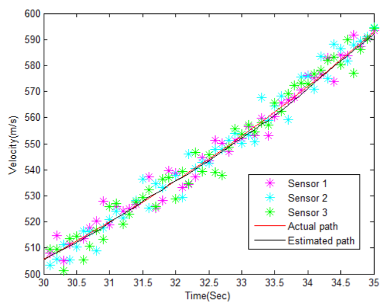
Where $D_{i,j}$ is the distance between projectile launch point to the intercepting points of the airborne object, $Z_{i,j}$ is the importance of the zone where missile may hit, $\vec{d}_i$ is the orientation of the airborne object trajectory, $\vec{d}_j$ is the orientation of the projectile trajectory, T_w is the reaction time of the projectiles, T_{wp} is the projectile preparation time and T_t is time for travelling the airborne object to the intercepting point after its detection. T_w and T_t are calculated in Section 2.2 and Section 2.3, respectively while T_{wp} is supplied by the domain Knowledge expert. $C_{i,j}$ in (12) denotes the cost value if jth projectile is assign to the ith airborne object. If cost value is less then projectile airborne object pairing is more perfect. So for all values of i and j, $C_{i,j}$ forms a matrix which is known as cost matrix. So for getting a optimum projectile airborne object pairing value of $C_{i,j}$ is used as an input to the Hungarian algorithm [5] of optimization. The algorithm gives a binary matrix as an output which tells the suitable projectile airborne object pairing. In the next section the results of the proposed algorithm is presented.

3. Results and Analysis

Simulation examples are used to test the proposed method. We assumed that the data are delivered by three sensors. We first simulate the trajectories of an airborne object and an interceptor using (1) and (11), respectively. As we assume that the inter-sensor noise is divergence in nature we simulate different sensor data by adding different amount of noises with the actual trajectory data. The noise is added by appending a Gaussian noise with a fixed amount of shift of the coordinate and velocity values of the actual trajectories. The Figure 2(b)-(d) shows that the Extended Hybrid Kalman filter based PHD filter proposed in Section 2.1 can successfully group the same airborne object and interceptor data delivered by different sensors. Figure 2(d) shows the thrust estimation of the PHD filter for airborne object.



(a)



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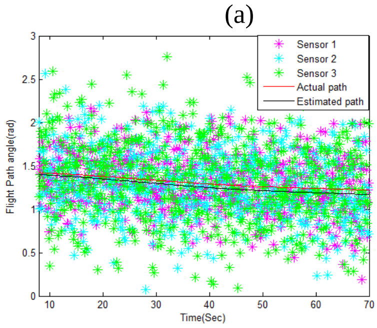

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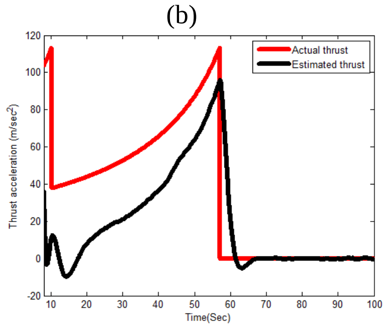
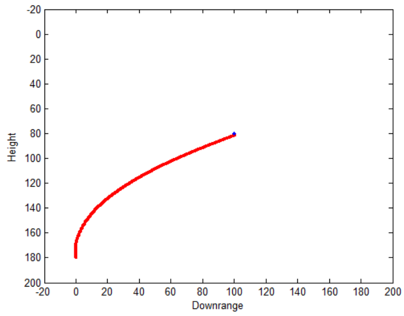


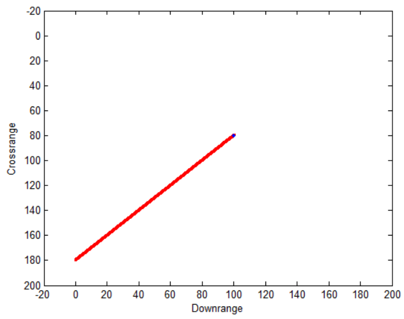
(d)

Figure 2. (a): The velocity value estimated from sensor data, actual velocity and estimated velocity, (b): Magnified part of the Figure 2(a), (c): Estimated flight path angle, (d): Thrust estimation patterns.

Figure 4 shows a typical interceptor simulation using (10) from front and top view positions respectively.



(a)



(b)

Figure 3. (a) Simulated projectile trajectory in front view, (b): Simulated projectile trajectory from the top view.

Finally in Table 1 and Table 2 we have shown an estimation and optimization of cost-matrix defined by (11). Figure 4 shows the top view of projectile airborne object pairing suggested in the Table 2. In Figure the read coloured lines represent six airborne objects mentioned in Table 1 and 2, while the green coloured lines represent four projectiles. The different shades in the figure represent degree of importance of the zones. The brighter zones are more important zone such as cities, houses, offices etc. while the darker zones are the less important zone such as agricultural fields, barren lands etc. Stars represent the launching points while the circles represent the hit points of the objects and projectiles. Figure 5 clearly shows that weapons are intercepting the missiles which try to hit the more important zones.

Table 1. A typical cost estimation using (11) for six airborne object and four projectiles scenario

	W1	W2	W3	W4
T1	.0364	.0596	.0571	.1503
T2	.0504	.0688	Inf	.0796
T3	1.3975	0.0665	Inf	.4283
T4	.2165	.1584	.1925	.0754
T5	.1385	.0435	Inf	.0846
T6	.0017	.0830	.2665	.0960

Table 2. Optimized solution of the cost matrix of Table 1.

	W1	W2	W3	W4
T1	0	0	1	0
T2	0	0	0	0
T3	0	0	0	0
T4	0	0	0	1
T5	0	1	0	0
T6	1	0	0	0

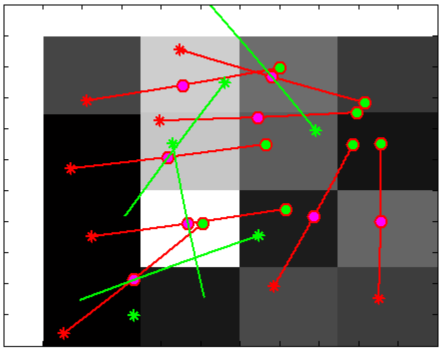


Figure 4: Graphical presentation of projectile airborne object paring evaluated in Table 2.

3.1 Comparison

The proposed work suggests a complete decision support system based on aeronautical safety system compared to only projectile assignment done in [6]. So our work brings tracking, fusion, parameter estimation, classification, path optimization and projectile airborne object pairing under single umbrella of a decision support system which is so far unique in nature.

4. Conclusion

We develop a decision support system for aeronautical safety system. The PHD filter, first order gradient-based optimization and Hungarian algorithm are exploited to build the system. The result proves the efficacy of our system. We will enhance our system by introducing versatility of PHD filter such as birth and spawn of different airborne objects and projectiles. Our study reveals that the partitioning of unique airborne object data from the multi-sensor data is most challenging in the situations where a projectile is very close to an airborne object or after interception when the airborne object and projectile spawn into multiple pieces. Fusing of such data is difficult as the values of the measurement parameters of different objects are very close to each other. We would address these issues in our subsequent works.

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